Queueing analysis of a battery-backed electric vehicle fast charging system

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ABSTRACT

We study a battery-buffered electric vehicle fast charging station modeled as a piecewise-deterministic Markov process. A battery buffer mediates between the grid connection and the charger, enabling fast service when charged and falling back to the grid rate when depleted. For the single-server loss system we derive the stationary distribution in closed form, uncovering a decoupling property whereby the charger state and battery level are independent in steady state, and obtain explicit formulas for the blocking probability, mean charging time, and a demand-dependent slowdown factor. We identify an optimal battery regime in which the average slowdown depends only on hardware parameters, yielding explicit battery sizing guidelines. We discuss the extension to s parallel chargers, showing that the interior balance equations reduce to an eigenvalue problem for a matrix pencil, and identify the boundary conditions that select the contributing modes. Simulation results are provided as examples.

Keywords

Piecewise deterministic Markov processes, Electrical Vehicles.

1. INTRODUCTION

The rapid adoption of electric vehicles (EVs) is placing increasing demands on electrical infrastructure. A key challenge is the provisioning of public *fast charging*: modern DC fast chargers can deliver 150 kW or more per port, requiring grid connections eventually sized for their peak output. Yet in practice a charging station is not used continuously; vehicles arrive stochastically, and the chargers may be idle a large portion of the time. Sizing the grid connection for worst-case simultaneous demand is therefore wasteful. Civil infrastructure investment becomes costly, and utility demand charges (fees based on peak power draw) can represent the dominant operating cost of such a facility [8]. A battery, acting as an energy buffer, placed between the grid and the chargers resolves this mismatch: the battery gradually charges from the grid at a steady, lower rate and discharges at high power during the brief intervals vehicles are present, enabling a much smaller and cheaper grid connection. While battery storage at EV stations has been studied before, it has been done primarily in the context of renewable energy integration, where the battery smooths intermittent solar or wind supply [1, 6, 10, 11]. However, the advent of fast DC chargers in highways and remote locations with limited grid access has made battery buffering a particularly compelling solution.

The central analytical question this raises is a queueing one: given that vehicles arrive randomly, and the battery recharges during idle or low utilization periods, how often does a vehicle arrive to find the battery depleted, forcing service at the slower grid rate? This coupling between the continuous battery state and the discrete nature of the vehicle queue is what distinguishes the battery-buffered station from a standard queue, and it has not been captured in the existing EV charging literature, where mostly fixed service rates are used [2, 5, 9].

In this paper, we propose a station model using a Piecewise-Deterministic Markov Process (PDMP) [3, 4]: the battery level evolves deterministically between random events, while car arrivals and departures trigger instantaneous state jumps. The state is then the pair (n, x) of system occupancy and battery level. We derive the generator of this Markov process, and compute the stationary distribution in closed form for the single-server loss system. A key structural finding is a *decoupling property*: in steady state the charger occupancy and battery level are conditionally independent given the battery is the interior of the state space. This enables to compute explicit formulas for the blocking probability, mean charging time, and a *slowdown factor* quantifying the service degradation as a function of a car's energy demand. Armed with these formulas, we discuss several performance issues, design objectives that provide guidelines on how to design such systems and provide some numerical examples.

We also discuss the extension of our analysis to the case where multiple chargers are available, where s fast-chargers share a common battery. In this case, the solution involves matrix-analytic methods and no closed form formulas can be obtained, but the problem remains numerically tractable.

2. SINGLE-SERVER MODEL

We consider a single EV charging station consisting of one charger connected to the electrical grid, equipped with a battery buffer of capacity $B > 0$. Cars arrive according to a Poisson process of rate $\lambda > 0$, each carrying an energy

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demand $Y \sim \mathrm{Exp}(\mu)$. The station operates as a *loss system*: a car that arrives when the charger is busy is turned away.

The system parameters are: the grid supplies a maximum power p_1, used to charge the battery or serve the car when present. When idle, the battery recharges at rate $0 < p_0 \leqslant p_1$; the inequality allows for slower recharging to preserve battery life. The charger delivers power $p_2 > p_1$ (fast charging), drawing p_1 from the grid and $c := p_2 - p_1 > 0$ from the battery. If the battery empties, the system reverts to charging from the grid alone at rate p_1, and remains so until the current car departs. We denote by $x(t) \in [0, B]$ the battery level and by $n(t) \in \{0, 1\}$ the presence of a charging vehicle.

When a car is admitted, service proceeds in at most two phases. In *Phase 1* (fast charging), while the battery level $x > 0$, the car charges at rate p_2 and the battery discharges at rate c. In *Phase 2* (slow charging), once the battery is exhausted, $x = 0$ and the car charges at rate p_1, while x remains at zero. A car with small demand may complete service entirely in Phase 1 if its service requirement is low enough. Under the exponential energy demands assumed before, the instantaneous departure rate from the system is therefore $\mu p(x)$, where:

$$p(x) = p_2 \, \mathbf{1}_{x>0} + p_1 \, \mathbf{1}_{x=0}. \tag{1}$$

The system state at time t is thus the pair $Z(t) = (n(t), x(t))$. Between arrival and departure events, the battery evolves deterministically according to $\dot{x} = r(n, x)$, where

$$r(n,x) = \begin{cases} p_0 & n = 0,\ x < B, \\ 0 & n = 0,\ x = B, \\ -c & n = 1,\ x > 0, \\ 0 & n = 1,\ x = 0. \end{cases} \tag{2}$$

The jumps are spontaneous events: *arrivals* at $(0, x)$ occur at rate λ and the states transitions (deterministically) to $(1, x)$; *departures* at $(1, x)$ with $x > 0$ occur at rate μp_2 and transition to $(0, x)$; *departures* at $(1, 0)$ occur at rate μp_1 and transition to $(0, 0)$, or in short $(1, x) \mapsto (0, x)$ at rate $\mu p(x)$. In all cases the battery level is unchanged at the jump epoch. A depiction of the typical cycle is given in Figure 1.

The process $Z(t)$ with state space $\{0, 1\} \times [0, B]$ fits the PDMP framework of Davis [3,4], though the standard theory requires an open state space and forces the process to jump before reaching the boundary. Neither condition holds here: E is compact, and the flow reaches $x = 0$ and $x = B$ in finite time without a jump. Nevertheless, the vector field (2) is bounded and monotone on each mode, so the flow is globally well-defined. The strong Markov property follows from the martingale-problem approach, which extends to this setting since all jump rates are bounded and the jump kernel leaves x unchanged (and thus the jumps are trivially bounded, as required by the theorem).

The extended generator $\mathcal{A}$ of $Z(t)$ has domain $\mathcal{D}(\mathcal{A})$ consisting of functions $\psi : E \to \mathbb{R}$ such that $\psi(n, \cdot)$ is absolutely continuous on $[0, B]$ for each n. For $\psi \in \mathcal{D}(\mathcal{A})$,

$$\mathcal{A}\psi(0,x) = r(0,x)\,\psi_0'(x) + \lambda\,[\psi(1,x) - \psi(0,x)], \tag{3a}$$

$$\mathcal{A}\psi(1,x) = r(1,x)\,\psi_1'(x) + \mu p(x)\,[\psi(0,x) - \psi(1,x)], \tag{3b}$$

where $\psi_n' = \partial_x \psi(n, \cdot)$ is defined Lebesgue-a.e. The first term in each line is the flow contribution; the second is the jump contribution, reflecting instantaneous mode switches at rate λ (arrivals) and $\mu p(x)$ (departures), leaving x unchanged. For any $\psi \in \mathcal{D}(\mathcal{A})$, the process $M_t^\psi := \psi(Z(t)) - \psi(Z(0)) - \int_0^t \mathcal{A}\psi(Z(s))\,ds$ is a local martingale [3], and the process is right-continuous and a strong Markov process.

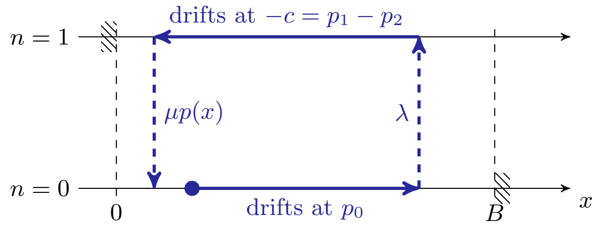

Figure 1: State space and system dynamics, showing a typical trajectory of $Z(t)$.

The adjoint operator $\mathcal{A}^*$ operates on the probability distributions on E and provides the forward Kolmogorov equation of the system [4]. A stationary distribution π satisfies $\int \mathcal{A}\psi\, d\pi = 0$ for all $\psi \in \mathcal{D}(\mathcal{A})$, which is the weak form of the Kolmogorov forward equation $\mathcal{A}^*\pi = 0$. We now proceed to solve the steady state of this system by positing a particular form of π using the characteristics of the system.

Note that an arriving car always finds $x > 0$ with probability one: during any idle period the battery charges at rate $p_0 > 0$, and since the idle duration is $\mathrm{Exp}(\lambda)$ (positive a.s.), the battery is strictly positive at every arrival epoch. Consequently, $(n = 0, x = 0)$ has zero probability under any stationary distribution, and we can restrict attention to $E' = \{0\} \times (0, B] \cup \{1\} \times [0, B]$.

3. SINGLE-SERVER IN STEADY-STATE

3.1 Stationary distribution

Since E is compact and $Z(t)$ is irreducible (every open rectangle $\{n\} \times (a, b)$ is reachable with positive probability from any starting point, by a combination of deterministic drift and Poisson events), the process is ergodic with a unique stationary distribution π [7].

We seek π as a mixed measure of the form:

$$\pi = f_0(x)dx + f_1(x)dx + \pi_{0,B}\delta_{(0,B)} + \pi_{1,0}\delta_{(1,0)}. \tag{4}$$

Here $f_n(x)$, are the interior densities and $\pi_{0,B}, \pi_{1,0} \geq 0$ are point masses at the corners where the flow accumulates: $x = B$ in mode 0 (since $r(0, \cdot) = p_0 > 0$) and $x = 0$ in mode 1 (since $r(1, \cdot) = -c < 0$). The other two corners should carry no atoms since flow arriving on them is immediately carried into the interior by the flow (cf. Figure 1).

Substituting this form into $\int \mathcal{A}\psi\, d\pi = 0$ and using eqs. (3), the stationarity condition $\int \mathcal{A}\psi\, d\pi = 0$ for all $\psi \in \mathcal{D}(\mathcal{A})$ yields the following balance equations for the interior densities:

$$p_0\, f_0'(x) = -\lambda\, f_0(x) + \mu p_2\, f_1(x), \tag{5a}$$

$$-c\, f_1'(x) = \lambda\, f_0(x) - \mu p_2\, f_1(x). \tag{5b}$$

As for the atoms, we obtain the following *boundary flux equations*, obtained by flux conservation at each boundary point. At $x = 0$ in mode 1, the inward flow $c\, f_1(0^+)$ feeds the atom $\pi_{1,0}$, which drains via departures at rate μp_1:

$$c\, f_1(0^+) = \mu p_1\, \pi_{1,0}. \tag{5c}$$

Similarly, at $x = B$ in mode 0, the inward flow $p_0 f_0(B^-)$ feeds the atom $\pi_{0,B}$, which drains via arrivals at rate λ:

$$p_0 f_0(B^-) = \lambda \pi_{0,B}. \tag{5d}$$

Finally, at $(n, x) = (0, 0)$, mass arriving from departures at rate μp_1 must immediately be carried at rate p_0 into the interior (no atom), yielding:

$$p_0 f_0(0^+) = \mu p_1 \pi_{1,0}. \tag{5e}$$

And finally by a similar argument, at $(n, x) = (1, B)$

$$c f_1(B^-) = \lambda \pi_{0,B}. \tag{5f}$$

Equations (5) provide the global balance equations for our system, which coupled with the normalization condition $\int_0^B (f_0 + f_1)\, dx + \pi_{0,B} + \pi_{1,0} = 1$ defines the steady-state distribution. We can now state the main result of this section.

THEOREM 3.1. *Let $\theta := \mu p_2/c - \lambda/p_0$. The unique stationary distribution of $Z(t)$ has the form (4) with*

$$f_0(x) = \frac{e^{\theta x}}{K}, \qquad f_1(x) = \frac{p_0}{c} \cdot \frac{e^{\theta x}}{K}, \qquad x \in (0, B), \tag{6a}$$

$$\pi_{1,0} = \frac{p_0}{\mu p_1 K}, \qquad \pi_{0,B} = \frac{p_0 e^{\theta B}}{\lambda K}, \tag{6b}$$

and the normalization constant is

$$K = \frac{p_0 + c}{c} \cdot \frac{e^{\theta B} - 1}{\theta} + \frac{p_0}{\mu p_1} + \frac{p_0 e^{\theta B}}{\lambda}, \tag{6c}$$

with the convention $(e^{\theta B} - 1)/\theta = B$ when $\theta = 0$.

PROOF. Adding (5a), (5b) gives $\frac{d}{dx}[p_0 f_0 - c f_1] = 0$, so $p_0 f_0 - c f_1$ is constant on $(0, B)$. Equations (5c) and (5e) together give $p_0 f_0(0^+) = c f_1(0^+)$, so the constant is zero. Therefore, $f_1(x) = (p_0/c) f_0(x)$ for all $x \in (0, B)$. Substituting this condition into (5a) yields a first order differential equation for f_0, which accounting for the definition of θ is simply $f_0'(x) = \theta f_0(x)$. We conclude that f_0 and f_1 are exponential with $f_0(x) = f_0(0^+) e^{\theta x}$. From this expression, the atoms follow directly from (5d), (5e), and (5f) is automatically satisfied, confirming consistency. Setting $f_0(0^+) = 1/K$ and imposing normalization determines K as in (6c). $\square$

The parameter θ has a physical interpretation: define the offered load of the system $\rho := \lambda/\mu$, i.e. the average external power request. Consider now a single idle-busy cycle in the absence of battery boundaries: the battery charges at rate p_0 during the idle period (mean duration $1/\lambda$) and discharges at rate c during the busy period with mean duration $1/(\mu p_2)$. Then, the expected net battery change per cycle is therefore $\Delta := \frac{p_0}{\lambda} - \frac{c}{\mu p_2}$. It is easy to see that θ has the same sign as Δ, in particular:

$$\theta > 0 \Leftrightarrow \rho < \frac{p_0 p_2}{c} = \frac{p_0 p_2}{p_2 - p_1}$$

This corresponds to a *lightly loaded* station, where the battery gains energy on average on each cycle and thus tends to fill, concentrating its mass towards $x = B$. When $\rho > p_0 p_2/c$, we are in the *highly loaded* regime, $\theta < 0$, the battery tends to drain and mass concentrates near $x = 0$, where a significant fraction of service occurs at the slow rate p_1. When $\theta = 0$ the battery neither drains nor fills on average and stationary density is uniform on $(0, B)$.

3.2 Performance metrics

We now derive key performance metrics directly from Theorem 3.1. We begin by computing the blocking probability, which, by the PASTA property, equals the stationary probability the charger is busy:

$$P_B = \int_0^B f_1(x)\, dx + \pi_{1,0} = \frac{p_0}{K} \left(\frac{e^{\theta B} - 1}{c\,\theta} + \frac{1}{\mu p_1} \right). \tag{7}$$

Note that this probability is also the probability that the charger is working. As $B \to 0$, $P_B \to \rho/(\rho + p_1)$, the blocking probability of an $M/M/1/1$ queue at rate p_1, since cars can only be served from the grid. Conversely, if the battery is large, as $B \to \infty$ we distinguish two cases: in the lightly loaded regime, $\theta > 0$ and $P_B \to \rho/(\rho + p_2)$, corresponding to service always at the fast rate p_2 because the battery can cope with the load and the system operates most of the time in fast charging. However, if the system is highly loaded ($\theta < 0$), P_B converges to a value strictly between these two limits even as $B \to \infty$, reflecting that the system operates *partly in both charging regimes* because the battery regularly depletes.

We now focus on the mean service time. Since there is at most one car in service, $E[N] = P_B$. By Little's law applied to admitted customers, $\lambda(1 - P_B)E[T] = P_B$, so the mean service time of an admitted car is:

$$\mathbb{E}[T] = \frac{P_B}{\lambda(1 - P_B)} = \frac{1}{1 + \frac{\lambda B}{p_0}} \left(\frac{e^{\theta B} - 1}{c\theta} + \frac{1}{\mu p_1} \right). \tag{8}$$

Another meaningful quantity is the *average slowdown*, which we define as $E[T]$ normalized by the average service time in a system that operates at the fast rate p_2 permanently, which is given by:

$$\bar{S} := \frac{\mu p_2\, \mathbb{E}[T]}{\mathbb{E}[Y]} = \frac{\mu p_2}{1 + \frac{\lambda B}{p_0}} \left(\frac{e^{\theta B} - 1}{c\theta} + \frac{1}{\mu p_1} \right). \tag{9}$$

A final quantity of interest is the *slowdown factor*, measuring service degradation relative to ideal fast charging as a function of requested energy:

$$S(y) := \frac{\mathbb{E}[T \mid Y = y]}{y/p_2} \geq 1. \tag{10}$$

The key quantity is $x_y := cy/p_2$, the battery level that would be exactly depleted by demand y: if admitted at $x \geq x_y$ the car completes entirely in fast charging; if $x < x_y$ the battery depletes mid-service and slow charging kicks in. This gives the compact formula

$$T(x, y) = \frac{y}{p_2} + \frac{(x_y - x)^+}{p_1}, \tag{11}$$

so that $S(y) = 1 + (p_2/(p_1 y))\, \mathbb{E}[(x_y - x)^+]$, where the expectation is over the admitted battery level. Note that even for moderate demands $y < x^*$, slowdown occurs whenever the car arrives at a battery level $x < x_y$. The threshold $x^* = p_2 B/c$ is the energy demand that would deplete even a full battery: for $y \geq x^*$, slow charging is unavoidable regardless of the battery level at admission.

PROPOSITION 3.2. *Let $x_y = cy/p_2$, $J(a) = (e^{\theta a} - 1 - \theta a)/\theta^2$, and $K(1 - P_B) = (e^{\theta B} - 1)/\theta + p_0 e^{\theta B}/\lambda$. Then:*

$$S(y) = 1 + \frac{p_2}{p_1\, y} \cdot \frac{J(x_y)}{K(1 - P_B)}, \quad y < \frac{p_2 B}{c}, \tag{12a}$$

$$S(y) = 1 + \frac{p_2}{p_1\, y}\left[\frac{J(B)}{K(1-P_B)} + (x_y - B)\right], \quad y \geq \frac{p_2 B}{c}. \tag{12b}$$

In both cases $S(y) \geq 1$, $S(y) \to 1$ as $y \to 0$ or $B \to \infty$, and $S(y) \to p_2/p_1$ as $y \to \infty$ or $B \to 0$.

The proof is based on applying the PASTA property to compute $\mathbb{E}[(x_y - x)^+]$ for an admitted car and is given in Appendix A.

3.3 Optimal battery regime

For practical battery health it is desirable to charge the battery *slowly*, and to keep it *cycling* continuously rather than maintaining it fully charged or fully depleted for long periods. The stationary distribution of Theorem 3.1 reveals a natural design criterion that achieves both goals simultaneously: namely control the value of the idle recharge rate p_0 to aim for the balanced regime where $\theta = 0$ and the battery is kept most of the time in $(0, B)$ and not the boundaries.

To analyze this, we first introduce the following definitions. Let $\alpha = p_2/p_1 > 1$ be the *speed-up ratio* of fast vs. slow charging. Let $\beta = \mu B p_2/c$ be the *normalized battery capacity*, the ratio between the battery drain time and the average fast recharge session duration, i.e. how many full fast-charge sessions the battery can sustain before depleting. Finally, define the *normalized load* $\tilde{\rho} = \rho/p_2 = \lambda/(\mu p_2)$.

We aim for the uniform stationary density achieved when $\theta = 0$. Setting $\theta = \mu p_2/c - \lambda/p_0 = 0$ uniquely determines the idle recharge rate as:

$$p_0^* = \frac{\lambda c}{\mu p_2} = \tilde{\rho}\, c. \tag{13}$$

The constraint $p_0^* \leqslant p_1$ requires then:

$$\tilde{\rho} \leqslant \frac{p_1}{c} = \frac{1}{\alpha - 1} \iff \rho \leqslant \rho_{\max} = \frac{p_1 p_2}{c}. \tag{14}$$

Below this threshold the operator can minimize battery stress by throttling down the idle recharge rate (slower recharge) and achieve the uniform regime. Above it, $\theta < 0$ regardless of p_0 and the battery drains in steady state.

Setting $\theta = 0$ and $p_0 = p_0^*$ in the expressions for K and P_B, one finds a simple form for the average slowdown in terms of the normalized battery capacity:

$$\bar{S} = \frac{\alpha + \beta}{\beta + 1}. \tag{15}$$

Remarkably, this depends only on α and β and not on the load. This is very useful from a design perspective since *the same design* works across all (feasible) loads $\rho < \rho_{\max}$, and decreases from $\alpha = p_2/p_1$ at $\beta \to 0$ (no battery) to 1 at $\beta \to \infty$. Setting a target slowdown $\bar{S}_0 \in (1, \alpha)$ we get:

$$\beta^* = \frac{\alpha - \bar{S}_0}{\bar{S}_0 - 1} \iff B^* = \frac{c}{\mu p_2}\frac{\alpha - \bar{S}_0}{\bar{S}_0 - 1}. \tag{16}$$

The required β^* grows with α: a large speed-up ratio means the battery drains fast relative to its recharge rate, demanding a larger buffer to keep the average slowdown modest. In the numerical example of Section 4, $\alpha = 2.5$ and $\bar{S}_0 = 1.2$ yield $\beta^* = 6.5$, i.e. the battery must sustain 6.5 mean fast-charge sessions before depleting — a conservative but necessary sizing given the aggressive speed-up ratio.

Finally, the blocking probability becomes:

$$P_B = \frac{\tilde{\rho}\bar{S}_0}{1 + \tilde{\rho}\bar{S}_0} = \frac{\tilde{\rho}(\alpha + \beta)}{\beta + 1 + \tilde{\rho}(\alpha + \beta)}. \tag{17}$$

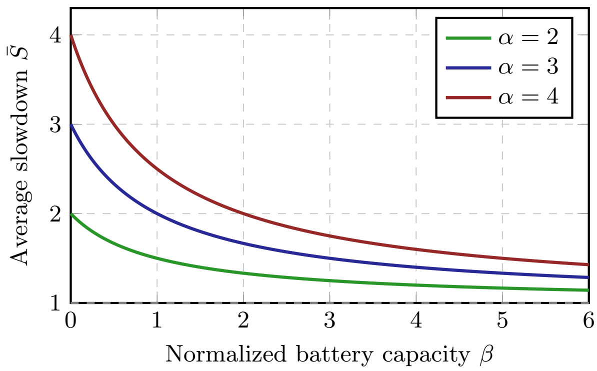

Figure 2: Average slowdown $\bar{S}$ in the optimal battery regime as a function of normalized capacity $\beta = \mu B p_2/c$, for three values of the speed-up ratio $\alpha = p_2/p_1$.

This has the form of an $M/M/1/1$ blocking formula with effective load $\tilde{\rho}\bar{S}_0$. Unlike $\bar{S}$, the blocking probability depends on $\tilde{\rho}$, as expected.

4. NUMERICAL RESULTS

We illustrate the theoretical results through event-driven simulations of the single-server system. We set $p_2 = 150\,\text{kW}$, a typical DC fast charger output, and $p_1 = 60\,\text{kW}$, representative of a medium-sized grid connection; this gives $c = 90\,\text{kW}$ and $\alpha = p_2/p_1 = 2.5$. For the energy demand distribution, large-scale data from the US Department of Energy reports an average of 22 kWh per paid DC fast charging session [8], consistent with a partial top-up rather than a full charge. We use $1/\mu = 20\,\text{kW h}$, giving a mean fast-charge session duration of $1/(\mu p_2) = 20/150 \approx 8\,\text{min}$.

We target an arrival rate of $\lambda = 3$ cars/h and aim to operate in the optimal battery regime ($\theta = 0$), which requires $p_0^* = \tilde{\rho} c = (3/7.5) \times 90 = 36\,\text{kW}$ and is feasible since $\tilde{\rho} = 0.4 < 1/(\alpha - 1) = 2/3$. We size the battery to achieve an average slowdown of $\bar{S}_0 = 1.2$, i.e. 20% above the ideal fast-charging time. From (16), $\beta^* = (\alpha - \bar{S}_0)/(\bar{S}_0 - 1) = 1.3/0.2 = 6.5$, giving $B^* = \beta^* c/(\mu p_2) = 6.5 \times 90/7.5 = 78\,\text{kW h}$. Note that a car with mean demand $20\,\text{kW h}$ charging at the slow rate $p_1 = 60\,\text{kW}$ would take $20\,\text{min}$; the design target of $\bar{S} = 1.2$ corresponds to a mean service time of $8 \times 1.2 = 9.6\,\text{min}$, a modest 1.6-minute penalty.

To illustrate the three battery regimes, we fix the design parameters and vary the arrival rate: $\lambda = 2$ (lightly loaded, $\theta > 0$), $\lambda = 3$ (critical, $\theta = 0$), and $\lambda = 4$ (heavily loaded, $\theta < 0$). Figure 3 shows a sample path of $x(t)$ for each case. For $\lambda = 2$ the battery drifts toward B and spends long periods near full; for $\lambda = 4$ it drains toward zero and service frequently degrades to the slow rate; for $\lambda = 3$ it oscillates freely across $(0, B)$ without dwelling at either boundary, consistent with the uniform stationary density predicted by Theorem 3.1.

In Figure 4 we plot the empirical stationary distribution of x for the critical case, showing good fit with the theoretical uniform density predicted by Theorem 3.1 at $\theta = 0$, plus the atoms at the boundaries. Finally, in Figure 5 we plot the theoretical slowdown curves $S(y)$ from eqs. (12) for the three load regimes, as well as an empirical estimation of the measured slowdown, using a kernel regression estimator to fit $E[T \mid Y = y]$ from the definition (10).

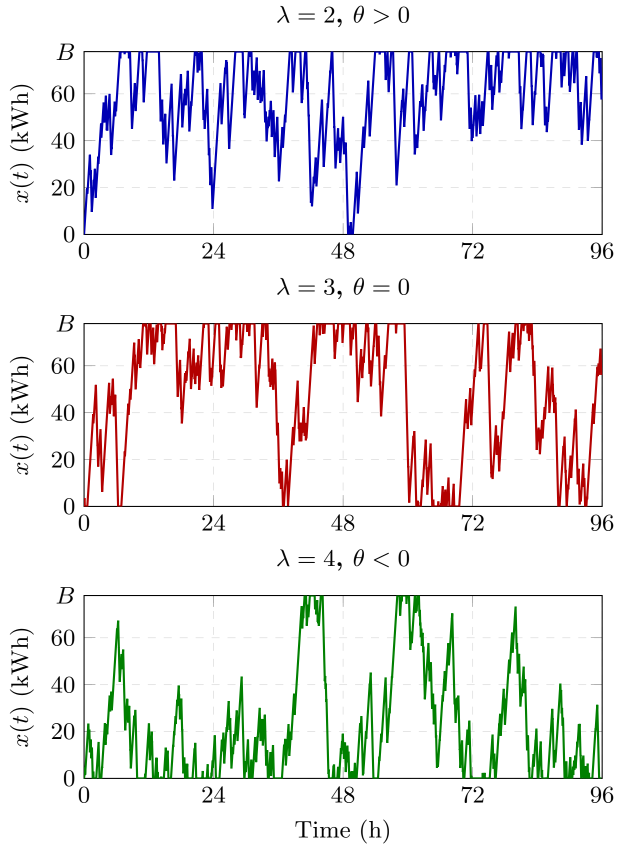

Figure 3: Sample battery trajectory for the three load regimes (lightly loaded, critical and heavily loaded).

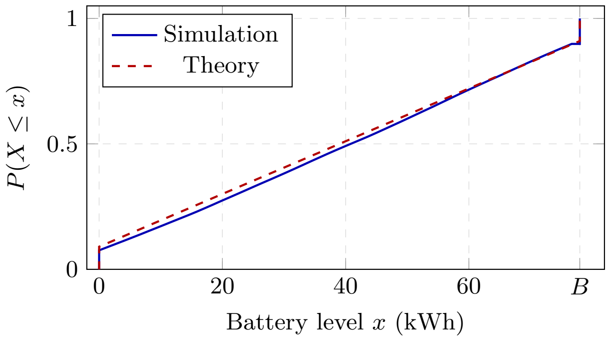

Figure 4: Stationary distribution of the battery level for $\lambda = 3$ ($\theta = 0$) including the atoms at $x = 0$ and $x = B$.

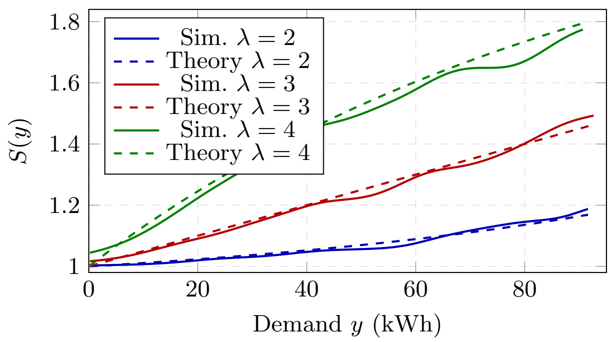

Figure 5: Slowdown factor $S(y) = \mathbb{E}[T \mid Y = y] \,/\, (y/p_2)$ as a function of demand y, for the three load regimes. Solid: kernel regression from simulation. Dashed: analytical formula.

5. THE MANY-SERVER CASE

The single-server analysis extends naturally to $s \geq 1$ parallel chargers sharing a battery buffer of capacity B and a grid connection of rate p_1, operating as a loss system. We set $p_0 = p_1$ for simplicity; the case $p_0 \neq p_1$ can be handled with additional notation.

The state is $Z(t) = (n(t), x(t))$ with $n(t) \in \{0, \dots, s\}$ and $x(t) \in [0, B]$. With $p_0 = p_1$ the net battery drift is $r_n = p_1 - np_2$ uniformly for all n: the battery charges when $n \leq n^* := \lfloor p_1/p_2 \rfloor$ and drains when $n > n^*$. Arrivals occur at rate λ when $n < s$; departures at total rate $n\mu p_2$ when $x > 0$ and at rate μp_1 when $x = 0$. The process is again an ergodic PDMP by the same arguments as Section 2.

The stationary balance equations on $(0, B)$ take the form $R\,\mathbf{f}'(x) = Q\,\mathbf{f}(x)$, where $\mathbf{f}(x) = (f_0(x), \dots, f_s(x))^T$, $R = \operatorname{diag}(r_0, \dots, r_s)$ is the diagonal drift matrix, and Q is the $(s + 1) \times (s + 1)$ transition rate matrix of the occupancy chain. When $p_2/p_1 \notin \mathbb{N}$, R is invertible and exponential solutions $\mathbf{f}(x) = \mathbf{a}\,e^{\theta x}$ require $(\theta, \mathbf{a})$ to be an eigenpair of $R^{-1}Q$. This pencil has $s + 1$ eigenvalues, one of which is always $\theta = 0$: setting $\theta = 0$ gives $Q\mathbf{a} = 0$, whose solution is the Erlang-B stationary distribution $a_n = \rho^n/n!$. The general interior solution is therefore a superposition of all $s + 1$ exponential modes.

The boundary conditions provide exactly $s + 1$ equations (two tridiagonal systems, one at $x = B$ and one at $x = 0$, coupled at $n = n^*$, plus normalization), which generically determine all $s + 1$ coefficients. In contrast with the single-server case, where the decoupling property of Theorem 3.1 reduces the solution to a single mode, the full spectrum of the pencil (Q, R) is generically needed for $s > 1$.

Two open questions must be resolved before a complete solution can be stated: whether the eigenvalues of $R^{-1}Q$ are always real, and whether the corresponding coefficients are always positive — both necessary for $\mathbf{f}(x)$ to be a valid density. We conjecture both hold, but proofs remain open. The complete matrix-analytic treatment is the subject of ongoing work.

6. CONCLUSIONS

We have studied a battery-buffered EV fast charging station using the framework of piecewise-deterministic Markov processes. For the single-server loss system we derived the stationary distribution in closed form, establishing a decoupling property that separates the battery level from the charger state in steady state, and obtained explicit expressions for the blocking probability, mean charging time, and slowdown factor. We identified a desirable battery regime in which the average slowdown depends only on hardware parameters and not on load, and derived explicit sizing formulas for the battery capacity and recharge rate.

The many-server extension reveals a richer structure where the interior density requires incorporating all the eigenmodes of the pencil (Q, R), and the boundary conditions couple two tridiagonal systems at each boundary. This matrix-analytic treatment is the subject of ongoing work.

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APPENDIX

A. PROOF OF PROPOSITION 3.2

From (11), $S(y) = 1+(p_2/(p_1y))\,\mathbb{E}[(x_y-x)^+]$, where x has the distribution of the battery seen by an admitted vehicle. By the PASTA property, this distribution is just the steady-state distribution conditioned on $n = 0$, i.e. has density $e^{\theta x}/(K(1-P_B))$ on $(0, B)$ and atom $\pi_{0,B}/(1-P_B)$ at $x = B$, with $\pi_{0,B} = p_0 e^{\theta B}/(\lambda K)$. Since $(x_y - x)^+ = 0$ for $x \geq x_y$, only admitted levels $x < x_y$ contribute. The fundamental integral needed is

$$J(a) := \int_0^a (a-x)\, e^{\theta x}\, dx = \frac{e^{\theta a} - 1 - \theta a}{\theta^2}, \tag{18}$$

obtained by integration by parts.

Case 1: $y < p_2B/c$. Here $x_y < B$, so the atom at $x = B$ satisfies $B > x_y$ and contributes nothing to $(x_y-x)^+$. Thus,

$$\mathbb{E}[(x_y - x)^+] = \frac{J(x_y)}{K(1-P_B)}.$$

Substituting $\mathbb{E}[(x_y - x)^+] = J(x_y)/K(1 - P_B)$ into $S(y) = 1 + (p_2/(p_1y))\mathbb{E}[(x_y - x)^+]$ gives (12a).

Case 2: $y \geq p_2B/c$. Here $x_y \geq B$, so all interior levels and the atom contribute. We compute

$$\begin{aligned}\int_0^B (x_y - x)e^{\theta x}\, dx &= x_y\frac{e^{\theta B}-1}{\theta} - \frac{Be^{\theta B}}{\theta} + \frac{e^{\theta B}-1}{\theta^2}\\ &= J(B) + (x_y - B)\frac{e^{\theta B}-1}{\theta},\end{aligned}$$

where the second equality uses $J(B) = (e^{\theta B} - 1 - \theta B)/\theta^2$. Adding the atom contribution $\pi_{0,B}(x_y-B) = (p_0e^{\theta B}/\lambda)(x_y-B)$:

$$\mathbb{E}[(x_y - x)^+] = \frac{J(B) + (x_y - B)\big[(e^{\theta B}-1)/\theta + p_0e^{\theta B}/\lambda\big]}{K(1-P_B)}.$$

Since $(e^{\theta B} - 1)/\theta + p_0e^{\theta B}/\lambda = K(1 - P_B)$, this simplifies to

$$\mathbb{E}[(x_y - x)^+] = \frac{J(B)}{K(1-P_B)} + (x_y - B).$$

Therefore,

$$S(y) = 1 + \frac{p_2}{p_1y}\left[\frac{J(B)}{K(1-P_B)} + (x_y - B)\right].$$

Since $p_2(x_y - B)/(p_1y) = c/p_1 - p_2B/(p_1y)$, we get $1 + c/p_1 - p_2B/(p_1y) = p_2/p_1 - p_2B/(p_1y)$, and substituting $J(B)$ yields (12b).

The limits follow directly: as $y \to 0$, $J(x_y) \sim \theta^2x_y^2/2 \to 0$ so $S \to 1$; as $y \to \infty$, Case 2 gives $S = p_2/p_1 - p_2B/(p_1y) + O(1/y) \to p_2/p_1$; as $B \to \infty$, $K(1 - P_B) \to \infty$ so $S \to 1$; as $B \to 0$, $x_y \geq B = 0$ always and $S = 1 + c/p_1 = p_2/p_1$. $\square$