Symmetrical components analysis for managing phase imbalance in EV charge scheduling

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ABSTRACT

This paper studies the scenario of an EV charging facility in which scheduling is used to manage the overall power consumption profile. We address the concern of a possible imbalance in the resulting 3-phase load, due to an uneven EV loading of the different phases. We apply symmetrical component analysis to develop metrics that capture this imbalance in a convex fashion in the natural problem variables, namely individual EV charging rates. We incorporate this metric into an online optimization for EV scheduling, and test in a scenario from the Caltech Adaptive Charging Network; simulations demonstrate this method is able to improve phase balance with a minimal impact on the service provided to the EV customers.

CCS CONCEPTS

• **Computer systems organization** → **Embedded systems**; *Redundancy*; Robotics; • **Networks** → Network reliability.

KEYWORDS

Phase imbalance, EV charging, Scheduling

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1 INTRODUCTION

The anticipated progression of Electrical Vehicles (EVs) to a predominant mode of transportation requires a parallel deployment of adequate charging facilities. In a distribution grid which is not greatly over-provisioned and where these new loads bring a significant increase, solutions will require a clever management of the scarce available capacity.

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We focus here on the situation of a centralized parking facility at a corporate or school site, which may provide a significant number of EV charging stations. A recent deployment of this nature is the Adaptive Charging Network developed at Caltech, see [6]. Of the many operational challenges that appear in this kind of installation, our aim is to contribute to the management of 3-phase *imbalance*; this arises because the presence of EVs in the facility is irregular, so the overall loading of phases will be typically uneven.

The tightest way to ensure approximate balance would be to endow each charging spot with switches and adequate cabling so that the facility could decide in real time which phase to load. We work here under the premise that this level of circuit redundancy is not admissible for economic reasons, and therefore each parking spot is preassigned a fixed single phase. One could think of trying to control the parking spot assignment upon EV arrival, but this leads into driver compliance issues which we would rather avoid; in its absence, can anything be done to mitigate power imbalance?

What is left is to affect the *schedule* of charging to take into account the imbalance issue. That is, to exploit the inherent flexibility in time of EV charging to even out the phase loading profile. Scheduling to reduce consumption peak has been considered in [6] and references therein, and the issue of imbalance was treated in [3, 5]. Our contribution here is to incorporate the method of symmetrical components [2], a standard way to analyze imbalance, to these online scheduling methods. In particular, we show how variants of standard metrics [1] can be incorporated in the form of convex penalties or convex constraints, thus allowing for its tractable inclusion in the scheduling algorithm.

The paper is organized as follows: in Section 2 we provide background on this kind of charging installations. Then in Section 3 we explain the analytical basis for the imbalance cost to include. An online optimization strategy that incorporates this metric is described in Section 4 and tested using the ACN simulation platform [7]. Conclusions are given in Section 5.

2 ELECTRICAL MODEL OF EV PARKING LOT

Consider an EV charging station with multiple charging spots or *Electrical Vehicle Supply Equipment (EVSE).* In typical installations, these EVSEs can be individual stations, paired chargers in a single mast in between two parking spots or so called "pods" with multiple charging points. These EVSEs are single phase, while in general due to the power requirements, the parking installation will be

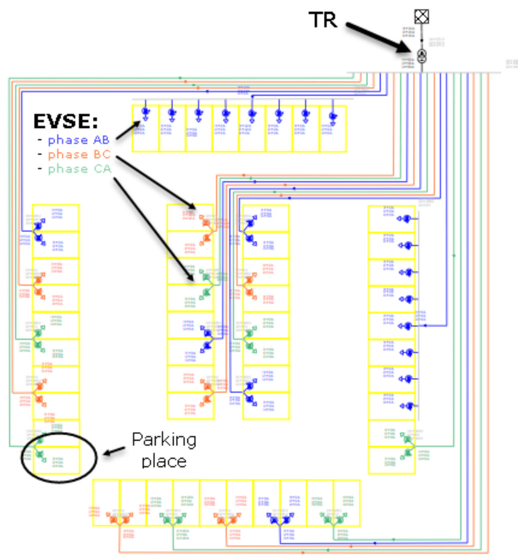

Figure 1: Electrical installation for an EV parking lot with charging stations. TR denotes the installation transformer and EVSE the different charging pods.

three-phase, and as such EVSEs will be connected in parallel, either between a pair of phases (Δ connection) or between a phase and the neutral wire (Y connection) depending on the installation.

As an example, consider the parking deployment depicted in Figure 1, which will be used later in the paper. In this case the system is connected to the grid through a step down transformer. The chargers (19 paired chargers and two 8-plug pods) are wired in parallel, each one of them in between two phases, denoted by AB, BC and CA as usual. Thus, in this case, the loads form a Δ configuration.

Connected EVs requiring charge will be plugged to one of the EVSEs, and therefore they will draw a current r_i from charger i. Since the distribution of vehicles is random and not controllable, and their charging state may be different across vehicles, this may lead to disparate current draws across the phases: an unbalanced system.

Phase imbalance poses several problems to the infrastructure: as an extreme example, if all loads appeared in single phase pair, the line current drawn from the transformer becomes a factor $\sqrt{3}$ higher than what would be in a balanced scenario. This leads to a design requirement of ≈ 70% increased size in the transformer just to handle this worst-case scenario. Moreover, working out of balance poses a significant strain on the transformer wiring and also on different parts of the cabling infrastructure of the parking lot and has to be taken into account in the charging constraints ([6]). Some regulation entities penalize clients for drawing out-of-balance currents from the network.

Several measures of imbalance are defined in industry standards [1]. Our main contribution is to show that, under a reasonable assumption, these metrics can be incorporated as convex penalties in the EV scheduling problem. Therefore, scheduling strategies can be appropriately modified to keep the system as balanced as possible.

3 QUANTIFYING IMBALANCE

As a charging station operator, the natural variable to control is the *charging rate* of each individual vehicle currently active. We denote by r_i the amount of (AC) current drawn by charger i. If we take into account all EVSEs connected in parallel to the same phase (say, phase a) of the 3-phase installation, we can write

$$\rho_a = \sum_{i \in a} r_i; \tag{1}$$

here $\rho_a = |I_a|$ is the magnitude of the current phasor I_a. Analogously we can define ρ_b, ρ_c for the remaining phases.

The above notation is directly applicable to Y-connected loads, where these currents coincide with the line currents drawn from the transformer. We treat this case first, and later on we extend to the situation (as in Figure 1) of a Δ configuration.

Due to the uneven distribution of load the magnitude of the phasors may be unequal, and also their phases could deviate from the ideal, 120^o positive sequence. However, since power is drawn from the grid which mostly imposes the voltage phases, and the power electronics involved in the charger impose a unit power factor differing only in the amount of current drawn, it is a reasonable approximation to assume that phases remain at 120^o and only the magnitudes differ. We shall see this assumption enables a tractable analysis.

Denoting $\alpha = e^{j2\pi/3}$ (note $\alpha^3 = 1$, $\overline{\alpha} = \alpha^2$), we state the following:

Assumption 1. *The current phasors I_a, I_b, I_c of the 3-phase system of EV loads satisfy*

$$I_a = \rho_a; \quad I_b = \rho_b \alpha^2; \quad I_c = \rho_c \alpha. \tag{2}$$

To study imbalance we use symmetrical component analysis (see e.g. [2], Ch 12.): this amounts to a change of coordinates from the original phasors to the positive sequence I^+, negative sequence I^- and zero-sequence I^0, defined by the Fortescue transfmormation:

$$\begin{bmatrix} I^0 \\ I^+ \\ I^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}. \tag{3}$$

Combining this formula with (2) leads to the expressions:

$$I^+ = \frac{1}{3}(\rho_a + \rho_b + \rho_c); \tag{4a}$$

$$I^- = \frac{1}{3}(\rho_a + \rho_b \alpha + \rho_c \alpha^2); \tag{4b}$$

$$I^0 = \frac{1}{3}(\rho_a + \rho_b \alpha^2 + \rho_c \alpha). \tag{4c}$$

We now express the magnitude square of the negative and zero components in terms of the variables of interest, in vector form $g := (\rho_a, \rho_b, \rho_c)^T$. After some calculations we obtain:

$$|I^-|^2 = |I^0|^2 = \frac{1}{9}\left(\rho_a^2 + \rho_b^2 + \rho_c^2 - \rho_a\rho_b - \rho_b\rho_c - \rho_c\rho_a\right),$$

a quadratic form which can be rewritten as:

$$|I^-|^2 = |I^0|^2 = \frac{1}{6} g^T P g = \frac{1}{6}\|Pg\|^2, \tag{5}$$

where $P = P^2 = P^T$ is the matrix

$$P = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & 1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}, \tag{6}$$

projection onto the orthogonal complement of $\mathbf{1} = (1,1,1)^T$. We have thus a convex quadratic penalty function for imbalance, which is zero in the span of $\mathbf{1}$, i.e. when loads are perfectly balanced. Alternatively, one may wish to use as imbalance metric the amplitude of negative (or zero) sequence components *relative* to the positive sequence amplitude $|I^+|$. This is more readily handled as a convex *constraint*: in particular, the condition

$$\frac{|I^-|}{|I^+|} \le \delta \tag{7}$$

is from (4) equivalent to the second order conic (SOC) constraint

$$\|Pg\| \le \frac{\delta\sqrt{6}}{3}\mathbf{1}^T g;$$

the same formula applies to the zero-sequence case.

Remark 1. *Metrics used in industry for imbalance can be found in the IEEE standard [1]; a first proposal is precisely the left-hand side of* (7). *A different metric attributed to ANSI is to consider deviations of each magnitude with respect to the average; this is connected to the approach above because one can readily show that* $Pg = g - \bar{g}\mathbf{1}$, *where* $\bar{g} = (\rho_a + \rho_a + \rho_a)/3$ *is the average of the load magnitudes.*

Yet another proposal (which applies only if the zero sequence component is zero), involves 4th moments of g; this will not be pursued here but it can be shown (in this case independently of Assumption 1) to be equivalent to a conic constraint on the magnitude squares.

We turn now to the situation of Δ-connected loads: i.e. chargers are connected between two phases a-b, b-c, and c-a; let I_{ab}, I_{bc}, and I_{ca} be the corresponding current phasors, which are taken to satisfy the analog of Assumption 1:

$$I_{ab} = \rho_{ab}; \quad I_{bc} = \rho_{bc}\alpha^2; \quad I_{ca} = \rho_{ca}\alpha. \tag{8}$$

From the point of view of the installation, it is the *line* currents which are subject to the balancing requirement: these are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} I_{ab} - I_{ca} \\ I_{bc} - I_{ab} \\ I_{ca} - I_{bc} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\alpha \\ -1 & \alpha^2 & 0 \\ 0 & -\alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} \rho_{ab} \\ \rho_{bc} \\ \rho_{ca} \end{bmatrix}. \tag{9}$$

Combining the above with the Fortescue transformation (3) yields

$$\begin{bmatrix} I^0 \\ I^+ \\ I^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 & 0 & 0 \\ 1-\alpha & 1-\alpha & 1-\alpha \\ 1-\alpha^2 & \alpha-1 & -\alpha+\alpha^2 \end{bmatrix} \begin{bmatrix} \rho_{ab} \\ \rho_{bc} \\ \rho_{ca} \end{bmatrix}. \tag{10}$$

The null zero-sequence component is consistent with the Δ load configuration. The positive sequence magnitude still follows (up to a factor $|1-\alpha| = \sqrt{3}$) the load average $\frac{1}{3}\mathbf{1}^T g$ as in (4a), where we now denote $g = (\rho_{ab}, \rho_{bc}, \rho_{ca})^T$.

For the negative sequence we have:

$$I^- = \frac{1-\alpha^2}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \end{bmatrix} g, \tag{11}$$

up to a factor analogous to (4b). We thus obtain the formula

$$|I^-|^2 = \frac{|1-\alpha^2|}{6} g^T P g = \frac{1}{2}\|Pg\|^2, \tag{12}$$

with P in (6). The conclusion is that, again in the Δ case, a measure of absolute imbalance can be expressed as a convex quadratic function of the vector g of load magnitudes and a constraint on relative imbalance can be expressed as a second-order cone in g.

4 SCHEDULING FOR SYSTEM BALANCE

We now apply the analysis of Sec. 3 to the parking installation of Fig. 1. Since EVSEs are wired in a Δ configuration, we use (12) to penalize imbalance in the scheduling optimization problem. We follow the Model Predictive Control (MPC) approach of [6], but note that (12) gives us an instantaneous penalty, so we formulate the problem as the following online optimization procedure.

Consider a discrete time setting with slots of duration δ. At a given point in time $t = 0$, the parking lot is occupied by vehicles requiring charge. Let us denote by $r_i(t)$ the charging rate of EVSE i during the slot t. Each vehicle has three main characteristics:

- $\bar{r}_i$, the maximum charging rate, with the convention $\bar{r}_i = 0$ if no vehicle is present at EVSE i.
- e_i, the remaining amount of charge needed to complete its demand.
- d_i, its departure time relative to the present.

Let now be $T = \max_i\{d_i\}$ the latest departure time for vehicles currently in the the system. We can formulate the following optimization problem:

Problem 1.

$$\max_{r_i(t)} \sum_{t=0}^{T-1} \left(\frac{T-t+1}{T}\right)\left[\sum_i r_i(t) - \beta g^T(t) P g(t)\right] \tag{13}$$

subject to:

$$0 \leqslant r_i(t) \leqslant \bar{r}_i \quad \forall i, t, \tag{14a}$$

$$\sum_{t=0}^{T} r_i(t)\delta \leqslant e_i, \tag{14b}$$

$$r_i(t) = 0 \; if \, t \geqslant d_i \quad \forall i \tag{14c}$$

$$g(t) = [\rho_{ab}(t), \rho_{bc}(t).\rho_{ca}(t)]^T \tag{14d}$$

Objective (13) tries to maximize delivered energy to all vehicles, with a penalty term on imbalance modulated by the tradeoff parameter β. A time-varying weight is added as in [6], to prioritize quick charge of the present vehicles in order to free up space for upcoming arrivals, since these are not directly considered in the online optimization.

Constraints (14a) and (14b) represent the power and energy constraints of the EVSEs and vehicles. In the energy constraint, AC current is multiplied by δ, the slot interval, and for simplicity we disregard AC-DC energy conversion factors, which can be appropriately incorporated. Constraint (14c) is a *presence* constraint, indicating that the vehicle can only be charged before its deadline expires. Additionally, some *infrastructure constraints* can be added to Problem 1. For instance, if a subset $\mathcal{I}$ of the EVSEs are connected

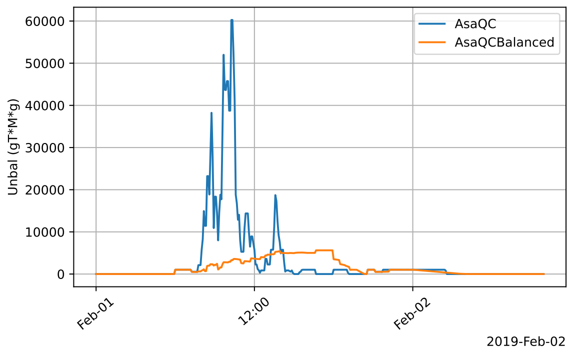

Figure 2: Balance term $g^T Pg$ evolution for the different scheduling problems.

on the same lines, then a total current constraint can be added as:

$$\rho_{\mathcal{I}}(t) = \sum_{i \in \mathcal{I}} r_i(t) \leqslant \bar{\rho}_{\mathcal{I}}.$$

Problem 1 yields the optimal schedule for the time horizon T, which is the latest departure of the current vehicles present. We now apply this strategy recursively following the MPC approach. Due to optimality, for any future time t, the optimal solution from $t, \ldots T$ is just the sub-sequence of the optimal schedule computed at time 0, we have to recompute the solution of Prob. 1 only when a new vehicle arrives. This is the algorithm we test below.

To evaluate the performance of our policy, we use data from [4] and take a typical day trace with arrival and departure information, as well as EVSE chosen by each vehicle and energy requested. We implemented our *balanced policy* in ACN-Sim. As a comparison standpoint, we choose the strategy from [6] named AsaQuickCharge (AsaQC) which corresponds to the same objective (13) taking $\beta = 0$.

We first plot in Fig. 2, the time evolution of the imbalance measure under the two algorithms, taking $\beta = 5 \times 10^{-3}$ for the balanced case (labeled AsaQC-Balanced). As expected, the balanced algorithm does a better job at keeping the total balance situation controlled by spreading out the charge of the vehicles.

We also show in Fig. 3 the moduli of the *line* currents I_a, I_b, I_c, for both algorithms. In the AsaQC case a large asymmetry appears between these currents, with peaks close to $400A$, indicative of more stress on the installation transformer. In contrast, the AsaQCBalanced algorithm keeps these currents relatively even and at much lower values.

To verify that balancing the system does not play against user experience, we also plot in Fig. 4 the total energy delivered by the algorithms. As we can see, the balanced system spreads out charge but delivers the same total amount of energy to the vehicles, thus satisfying demand completely.

Finally, it is important to validate Assumption 1, in its version (8) for Δ-connected loads. For this purpose, we simulate the schedule generated by our balanced algorithm in SinCal [8], a simulator of the entire electrical system. Fig. 1 shows the changes in the angle of voltage phasors for each phase-pair, with respect to the base case with no load. With unit power factor loads, analogous deviations

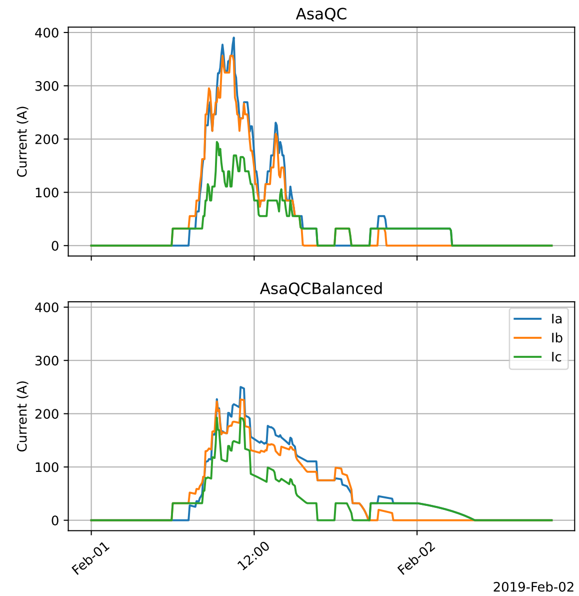

Figure 3: Current moduli evolution in the three phases.

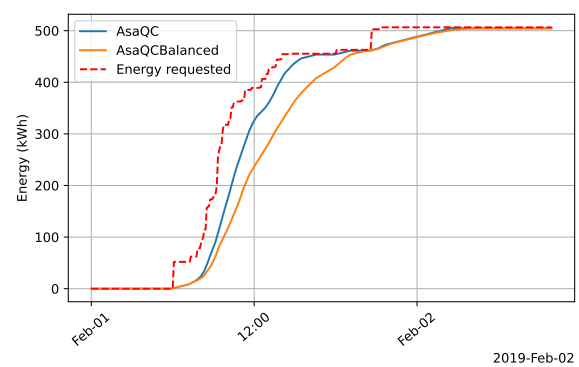

Figure 4: Total energy delivered.

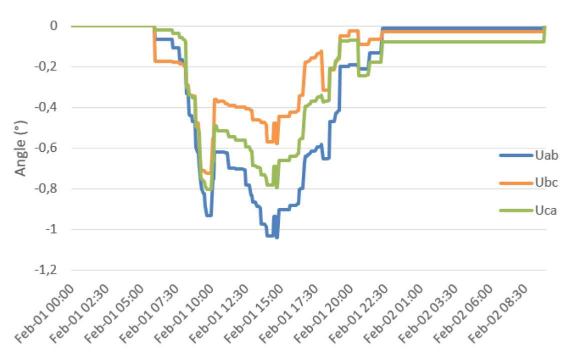

Figure 5: Angle deviations for each phase.

would hold for the angles of the current phasors. We see that each phase angle is altered by at most 1^o, and the relative deviation changes by less than 0.5^o, so phasors are kept approximately at 120^o difference at all times, validating the approach.

5 CONCLUSIONS

In this work, we proposed using the symmetrical components method to generate a tractable measure of phase imbalance. This measure is convex on the problem variables, enabling us to extend the methods of [6] to exploit user flexibility in charging time to keep the system appropriately balanced, which poses less strain on the infrastructure. Simulations show that the algorithm keeps the system balanced, particularly at high load, without hindering user perceived performance.

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