Spatial load balancing for EV charging customers

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Introduction

- Electrical vehicle (EV) adoption is currently growing exponentially.
- Less carbon emissions, noise and other efficiency benefits.

Problems:

- Charging is power and energy intensive for the network.
- Still, charging requires a lot of time.
- We need to build a charging infrastructure to replace gas stations.

Introduction

Some open questions...

- How to estimate charging demand in a region.
- How to design the public charging infrastructure.
- How to design incentives for users to reduce network congestion.
- What can we do to exploit flexibility in user demand to reduce power draw.

Outline of the talk

Spatial load balancing

Dynamic queueing model

Stability results

Simulations

Estimating spatially distributed demand

Final remarks

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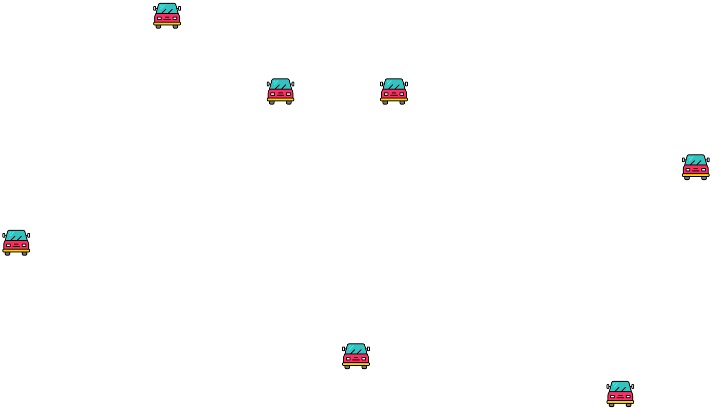
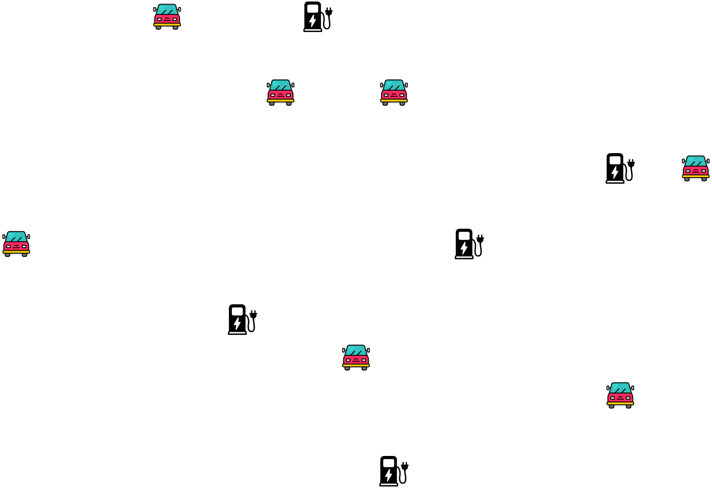
Final remarks

Spatial demand problem



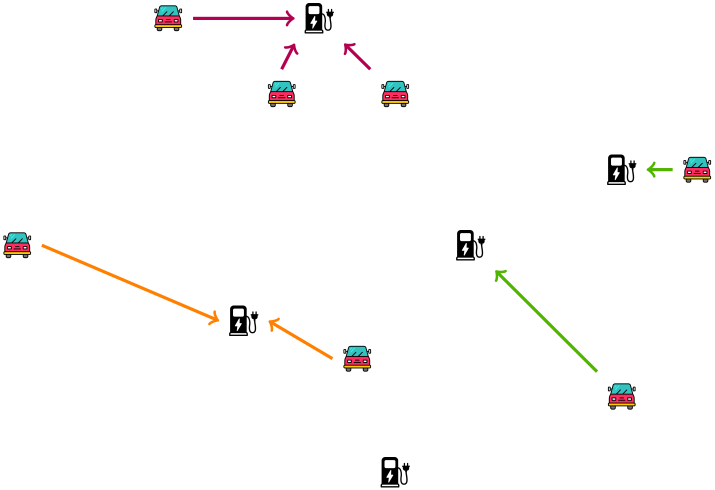
Spatially distributed requests

Spatial demand problem



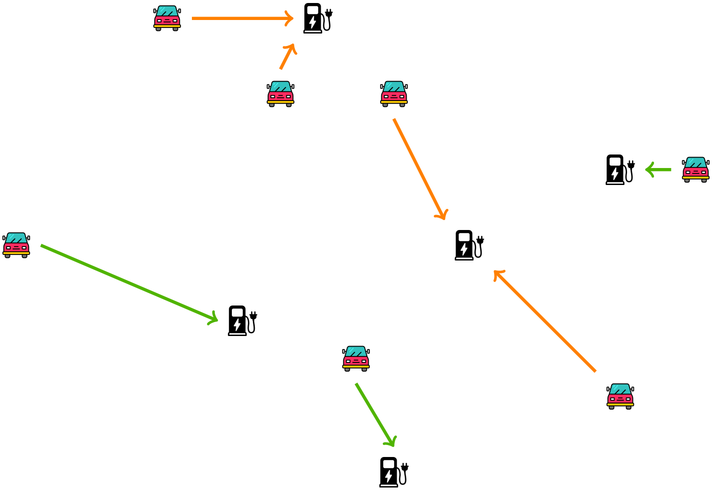
Spatially distributed servers

Spatial demand problem



Minimum distance assignment, does it make sense?

Spatial demand problem



Maybe it's better to balance more evenly...

Classical assignment problem

Define:

- $\bar{q}_i$ cars arrive at location i.
- There are $\bar{s}_j$ chargers at location j.
- C is the *cost matrix*, $c_{ij} \geqslant 0$ is the cost of getting from $i \to j$.

Classical assignment problem

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- $\bar{q}_i$ cars arrive at location i.
- There are $\bar{s}_j$ chargers at location j.
- C is the *cost matrix*, $c_{ij} \geqslant 0$ is the cost of getting from $i \to j$.

Then one would like to solve:

$$\min \sum_{i,j} c_{ij}\gamma_{ij},$$

subject to:

$$\sum_j \gamma_{ij} = \bar{q}_i, \quad \forall i,$$

$$\sum_i \gamma_{ij} = \bar{s}_j, \quad \forall j,$$

$$\gamma_{ij} \in \{0, 1\}.$$

Classical assignment problem

Monge transport problem, Kantorovich relaxation...

It is easier to relax the integrality constraint:

$$\min_{\Pi \geqslant 0} \sum_{i,j} c_{ij} \pi_{ij},$$

subject to:

$$\sum_j \pi_{ij} = \bar{q}_i, \quad \forall i,$$
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Relaxation is exact if quantities are integers... [Paganini et al CDC 2022]

Classical assignment problem

A simple example...

Assume $\bar{q}_i \equiv 1$ and $\bar{s}_j \equiv 1$, and the number of cars and chargers is the same, then:

$$\min_{\Pi \geqslant 0} \sum_{i,j} c_{ij} \pi_{ij},$$

subject to:

$$\sum_i \pi_{ij} = 1 \quad \forall j, \quad \sum_j \pi_{ij} = 1, \quad \forall i$$

i.e. Π is a doubly stochastic matrix.

The vertices of the feasibility region are exactly the permutation matrices (Birkhoff theorem), so the solution is integer valued.

Allocation problem

If instead supply is abundant ($\sum_j \bar{s}_j \geqslant \sum_i \bar{q}_i$) then we can formulate:

Problem 1

$$\min_{\Pi \geqslant 0} \sum_{i,j} c_{ij} \pi_{ij},$$

subject to:

$$\sum_j \pi_{ij} = \bar{q}_i, \quad \forall i,$$
$$\sum_i \pi_{ij} \leqslant \bar{s}_j, \quad \forall j.$$

Dual formulation

Let's write the Lagrangian of Problem 1:

$$L_1(\Pi, \mu) = \sum_{i,j} c_{ij}\pi_{ij} + \sum_j \mu_j \left[\sum_i \pi_{ij} - \bar{s}_j\right] = \sum_{i,j}(c_{ij} + \mu_j)\pi_{ij} - \sum_j \mu_j \bar{s}_j.$$

Now minimize over Π subject to $\sum_j \pi_{ij} = \bar{q}_i$:

$$\min_{\Pi} \sum_{i,j}(c_{ij} + \mu_j)\pi_{ij} - \sum_j \mu_j \bar{s}_j,$$

subject to:

$$\sum_j \pi_{ij} = \bar{q}_i \quad \forall i.$$

Remark: the problem decouples over i.

Solution structure of Problem 1

For each i, send the "mass" $\bar{q}_i$ from location i to the "cheapest" station using the modified cost $c_{ij} + \mu_j$.

- μ_j naturally acts as a congestion price on station j.

Solution structure of Problem 1

For each i, send the "mass" $\bar{q}_i$ from location i to the "cheapest" station using the modified cost $c_{ij} + \mu_j$.

- μ_j naturally acts as a congestion price on station j.

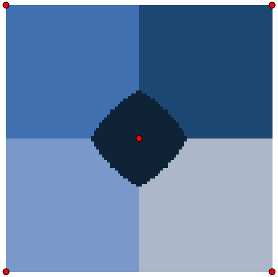
Example: Let c_{ij} be the Euclidean distance then:

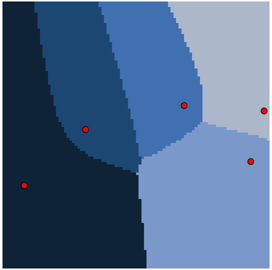
- If supply $\bar{s}_j$ is abundant, $\mu_j \equiv 0$ and the routing splits traffic along Voronoï cells.
- When congestion occurs, $\mu_j > 0$ and the attraction region of station j is such that:

$$d(x, y_j) + \mu_j \leqslant d(x, y_k) + \mu_k \quad \forall k \neq j$$

Examples

Gaussian distribution for demand, centered at the origin.





Dual function

Define now:

$$\gamma_i(C,\mu) = \arg\min_{z\in\Delta} \sum_{j=1}^{n} (c_{ij} + \mu_j) z_j.$$

where Δ is the unit simplex, then:

$$\pi^*_{ij}(C,\mu) = \bar{q}_i \gamma_{ij}(C,\mu),$$

and

$$D_1(\mu) = \sum_i \bar{q}_i \min_j (c_{ij} + \mu_j) - \sum_j \mu_j \bar{s}_j,$$

piecewise-linear and concave, with super-gradient

$$\partial_j D_1 = \sum_i \bar{q}_i \gamma_{ij}(c_{ij} + \mu_j) - \bar{s}_j.$$

Explicit congestion penalty

We may also use a soft constraint on $\bar{s}_j$:

Problem 2

$$\min_{\Pi \geqslant 0, s \geqslant 0} \sum_{i,j} c_{ij}\pi_{ij} + \sum_j \phi_j(s_j),$$

subject to:

$$\sum_j \pi_{ij} = \bar{q}_i, \quad \forall i,$$
$$\sum_i \pi_{ij} = s_j, \quad \forall j.$$

Here $\phi_j(\cdot)$, increasing and convex, $\phi_j(0) = 0$, measures congestion at node j.

Explicit congestion

Lagrangian duality

Problem 2 is always feasible, and extends the range to $s_j > \bar{s}_j$ which amounts to waiting for service. Its Lagrangian is:

$$\begin{aligned} L_2(\Pi, s, \mu) &= \sum_{i,j} c_{ij}\pi_{ij} + \sum_j \phi_j(s_j) + \sum_j \mu_j \Big[\sum_i \pi_{ij} - s_j \Big] \\ &= \sum_{i,j} (c_{ij} + \mu_j)\pi_{ij} + \sum_j \left[\phi_j(s_j) - \mu_j s_j\right]. \end{aligned}$$

Explicit congestion

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&= \sum_{i,j} (c_{ij} + \mu_j)\pi_{ij} + \sum_j \left[\phi_j(s_j) - \mu_j s_j \right].
\end{aligned}
$$

- The minimum over Π is the same as before.
- The minimum over s is the negative of the Fenchel transform of ϕ:

$$
\phi_j^*(\mu_j) = \min_{s_j} \{ \mu_j s_j - \phi_j(s_j) \}.
$$

Dual function

The dual function becomes:

$$D_2(\mu) = \sum_i \bar{q}_i \min_j (c_{ij} + \mu_j) - \sum_j \phi_j^*(\mu_j).$$

If ϕ_j is increasing and differentiable, then the optimum should satisfy:

$$\phi_j'(s_j^*) = \mu_j,$$

and we have the following gradient:

$$\partial_j D_2(\mu) = \sum_i \bar{q}_i \gamma_{ij}(c_{ij} + \mu_j) - [\phi_j']^{-1}(\mu_j).$$

Outline of the talk

Dynamic load balancing

The previous analysis considered a “one-shot” allocation, but in practice:

- Requests will arrive asynchronously.
- The system should generate incentive compatible congestion signals.
- If demand is stationary, the dynamics should converge to an equilibrium consistent with what a central planner would do.

Here is when queueing theory and fluid models come into play!

System model

- Each location i receives a flow of requests at rate $r_i(t)$ requests/sec.
- The load balancing rule (to be designed) splits input flows into rates $r_{ij}(t)$ from location i to station j.
- The total flow rate into station j is thus $a_j := \sum_i r_{ij}$.

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- The load balancing rule (to be designed) splits input flows into rates $r_{ij}(t)$ from location i to station j.
- The total flow rate into station j is thus $a_j := \sum_i r_{ij}$.
- The state variable $s_j(t)$ reflects the current assignment of station j; if it exceeds capacity $\bar{s}_j$, we allow the possibility of waiting for service.
- The state evolution corresponds to the fluid queue

$$\dot{s}_j = a_j - d_j(s_j),$$

where d_j is the departure rate from station j, $d_j(0) = 0$.

System model

Adding congestion signals

Key idea

Use time as the common ground for transportation and congestion costs.

System model

Adding congestion signals

Key idea

Use time as the common ground for transportation and congestion costs.

- Let c_{ij} be the transport time from $i \to j$ (e.g. $\frac{1}{\nu} d(x_i, y_j)$)
- Let μ_j be the waiting time at server j.
- Then $c_{ij} + \mu_j$ is the total delay before service.
- Selfish drivers then split according to

$$r_{ij}(t) = r_i \gamma_{ij}(C, \mu(t))$$

with γ_{ij} as before.

System model
Fluid queue model

A natural fluid model for the *waiting time* would be:

$$\mu_j := \frac{[s_j - \bar{s}_j]^+}{d_j(s_j)}$$

- $[s_j - \bar{s}_j]^+$ is the excess number of requests at station j, i.e. the queue.
- Dividing by the departure rate we get the time to get service.

System dynamics

The complete dynamics in the occupation state varible s_j are then:

$$\dot{s}_j = \sum_i r_i \gamma_{ij}(C, \mu) - d_j(s_j),$$

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Remark: We still need to model the departure rate. We developed two different choices:

- Sojourn-time model $\rightarrow$ customers have fixed sojourn times.
- Service-time model $\rightarrow$ customers have fixed service requirements.

Outline of the talk

Sojourn time model

- Assume users have a *time budget* that must be split between reaching the station, waiting for service and getting service.
- In order to maximize service time, selfish users will seek to minimize $c_{ij} + \mu_j$.
- If T is the average sojourn time, the departure rate is:

$$d_j(t) = \frac{1}{T} s_j(t),$$

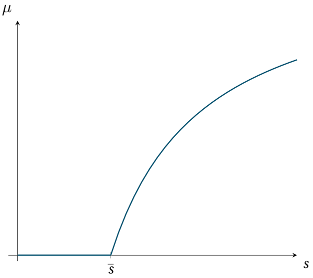
- The queueing delay (congestion price) becomes:

$$\mu_j(s_j) = \frac{[s_j - \bar{s}_j]^+}{d_j(s_j)} = T\left[1 - \frac{\bar{s}_j}{s_j}\right]^+.$$

Sojourn time model

Congestion price:

$$\mu_j(s_j) = T\left[1 - \frac{\bar{s}_j}{s_j}\right]^+.$$



Sojourn time model

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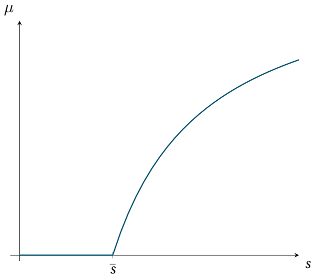
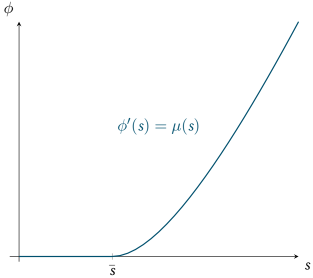


Penalty function:

$$\phi_j(s_j) := \begin{cases} 0 & s_j \leq \bar{s}_j, \\ T\left(s_j - \bar{s}_j + \bar{s}_j \log\left(\frac{\bar{s}_j}{s_j}\right)\right) & s_j > \bar{s}_j. \end{cases}$$



Equilibrium

Let $\bar{q}_i = r_i T$ be the steady state number of customers in the system from location i, then:

Theorem

The following are equivalent:

(a) s^* *is an equilibrium point of the dynamics. In particular, with* $\mu_j^* = \phi_j'(s_j^*)$ *there exists a split* $\gamma^* \in \gamma(c, \mu^*)$ *such that* $a_j^* = \sum_i r_i \gamma_{ij}^* = d_j(s_j^*)$. *Set* $\pi_{ij}^* = \bar{q}_i \gamma_{ij}^*$.

(b) (π^*, s^*, μ^*) *is a saddle point of the Lagrangian* L_2 *of Problem 2.*

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(b) (π^*, s^*, μ^*) *is a saddle point of the Lagrangian* L_2 *of Problem 2.*

So the distributed load balancing equilibrium solves:

$$\min_{\Pi \geqslant 0} \sum_{i,j} c_{ij} \pi_{ij} + \sum_j \phi_j(s_j),$$

with the preceding barrier function.

Stability

Theorem

The dual function $D_2(\mu)$ for Problem 2 is non-decreasing along trajectories of $\mu(t)$ arising from the sojourn-time dynamics.

Proof:

$$\frac{d}{dt} D_2(\mu(t)) = \sum_j \frac{\partial D_2}{\partial \mu_j} \dot{\mu}_j = \sum_{j \in \mathcal{J}} \underbrace{\left[\sum_i r_i T \gamma_j (c_{ij} + \mu_j) - [\phi_j']^{-1}(\mu_j) \right]}_{= T a_j - s_j} \dot{\mu}_j$$

$$= \sum_j \underbrace{[T a_j - s_j)]}_{= T \dot{s}_j} T \frac{\bar{s}_j}{s_j^2} \dot{s}_j = \sum_{j \in \mathcal{J}} \bar{s}_j \left(\frac{T \dot{s}_j}{s_j} \right)^2 \geqslant 0.$$

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Combined with a LaSalle type of argument, we get asymptotic stability.

Service time model

- Assume users arrive at location i with rate r_i.
- They have a *service requirement*, with average service time T_0.
- The number of tasks in service at station j is $\min\{s_j, \bar{s}_j\}$ so the departure rate is:

$$d_j(t) = \frac{\min\{s_j, \bar{s}_j\}}{T_0}.$$

- The congestion signal becomes:

$$\mu_j(s_j) = \frac{[s_j - \bar{s}_j]^+}{d_j(s_j)} = T_0 \left[\frac{s_j}{\bar{s}_j} - 1\right]^+.$$

The complete dynamics are:

$$\dot{s}_j = \sum_i r_i \gamma_{ij} - d_j,$$

$$d_j = \frac{1}{T_0} \min\{s_j, \bar{s}_j\},$$

$$\mu_j = T_0 \left[\frac{s_j}{\bar{s}_j} - 1 \right]^+,$$

$$\gamma_{ij} = \arg\min_j \{c_{ij} + \mu_j\}$$

Now the total service rate available is $\sum_j \bar{s}_j$.

So the natural stability condition is:

$$\sum_i \underbrace{r_i T_0}_{\bar{q}_i} \leqslant \sum_j \bar{s}_j$$

Service time model

Stability condition

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So the natural stability condition is:

$$\sum_i \underbrace{r_i T_0}_{\bar{q}_i} \leqslant \sum_j \bar{s}_j$$

This is *exactly* Problem 1 feasibility condition!

Service time model

Equilibrium

Theorem

Let $\bar{q}_i = r_i T_0$ and that the stability condition holds, then:

(a) *If s^* is an equilibrium point of the dynamics, there exists a choice of split $\gamma^* \in \gamma(c, \mu^*)$ such that $a_j^* = \sum_i r_i \gamma_{ij}^* = d_j(s_j^*)$. Set $\pi_{ij}^* = \bar{q}_i \gamma_{ij}^*$; then (π^*, μ^*) is a saddle point of the Lagrangian L_1.*

(b) *Let (π^*, μ^*) be a saddle point of the Lagrangian L_1 of Problem 1. Define:*

$$s_j^* = \begin{cases} \sum_i \pi_{ij}^* & \text{if } \mu_j^* = 0; \\ \bar{s}_j \left(1 + \frac{\mu_j}{T_0}\right) & \text{if } \mu_j^* > 0; \end{cases}$$

Then s^* is an equlibrium of the dynamics.

Service time model

Equilibrium

Remark: Note the difference between the equilibrium occupation s^* per station, and the allocation of Problem 1, $\hat{s}$. The latter will always be bounded by capacity, whereas the former will exceed it at congested stations.

Remark: Part (b) requires condition $\sum_j \bar{s}_j \geq \sum_i \bar{q}_i$ for feasibility of Problem 1 and therefore existence of equilibria. Substituting $\bar{q}_i = r_i T_0$ we arrive at the stability condition stated prior to the theorem.

Service time model

Stability

The dynamic analysis in this case strongly parallels that of the previous problem.

Proposition

The dual function $D_1(\mu)$ for Problem 1 with $\bar{q}_i = r_i T_0$ is non-decreasing along trajectories.

Proof:

$$\frac{d}{dt} D_1(\mu(t)) = \sum_j \frac{\partial D_2}{\partial \mu_j} \dot{\mu}_j = \sum_j \underbrace{\left[\sum_i r_i T \gamma_j (c_{ij} + \mu_j) - \bar{s}_j \right]}_{= T_0 a_j - s_j} \dot{\mu}_j$$

$$= \sum_{j \in \mathcal{J}} [T_0 a_j - \bar{s}_j] \frac{T_0}{\bar{s}} [a_j - \bar{s}_j] = \sum_{j \in \mathcal{J}} \frac{T_0^2}{\bar{s}_j} (\dot{s}_j)^2 \geq 0;$$

Outline of the talk

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Simulations

Estimating spatially distributed demand

Final remarks

Simulations

We now present some simulations for the sojourn time model:

- Cars arrive as a Poisson process of intensity $r = 4$ EV/min. in the square $[0, 1] \times [0, 1]$.
- Spatial distribution is a centered Gaussian.
- Exponential sojourn times with $T = 60$ min.
- Average number of customers: $rT = 240$.
- Car speed is $v = 0.1$, so the maximum travel time is ≈ 7 min.

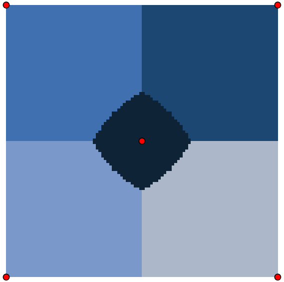
- Stations located at the center of the square and on the vertices.
- $\bar{s}_j \equiv 60$ amounts to 300 chargers.
- The central location becomes congested due to high demand in the region.

t

- Problem 2 equilibrium:

$$s_1^* = 64.4, \quad s_j^* = 42.4, \, j = 2, \ldots, 5,$$

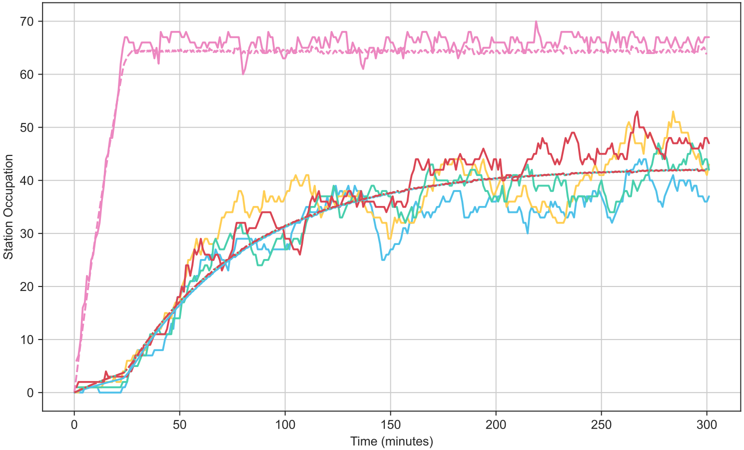
and the congestion price $\mu_1 = 4.1$ minutes.



Attraction regions

Simulations

Example 1



Time evolution for the stochastic system. Solid lines are the ODE.

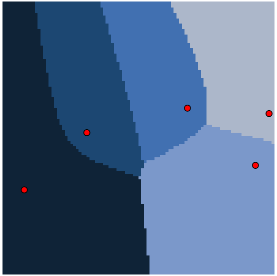
Simulations

Example 2: randomly chosen locations

- Again 5 stations, but located at random points in the square.
- Rest of the parameters as before.
- Problem 2 equilibrium:

$$s_1^* = 48.9,\ s_2^* = 61.8,\ s_3^* = 61.6,$$
$$s_4^* = 49.5,\ s_5^* = 12.3.$$

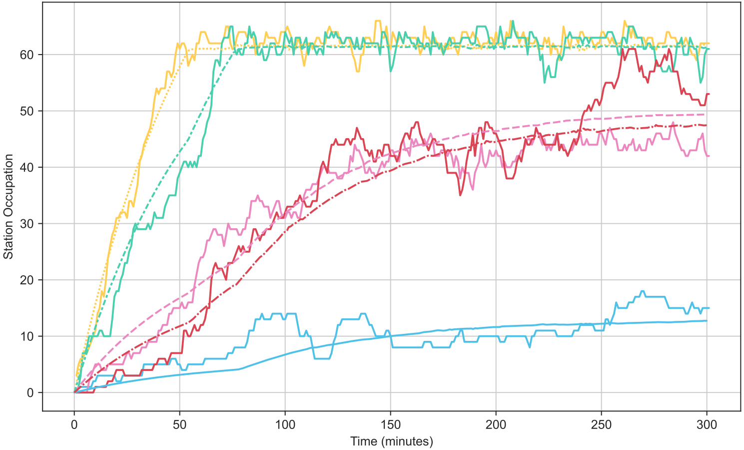
- Two stations become congested (the ones near the center of the region). $\mu_2^* \approx 1.7$ and $\mu_3^* \approx 1.5$ according to Problem 2.



Attraction regions

Simulations

Example 2: randomly chosen locations



Time evolution for the stochastic system. Solid lines are the ODE.

Open questions

- Space is discretized in our model, but it works great for distributed arrivals.
- Show convergence of the underlying stochastic model to the fluid limit.
  - Should be farly straightforward, but devil is in the details...
- Diffusion approximations around equilibrium?
  - Switching equations pose an issue (the $\min\{s_j, \bar{s}_j\}$ term)...
- Consider abandonments, incoming vehicles, etc. etc.

Outline of the talk

Spatial estimation

Ongoing work...

Motivation:

- We would like to know how much energy will EV cars need and where.
- Idea: use current consumption measures at gas stations.
- Convert to kilometers and then to energy using efficiencies.

Spatial estimation

Main problem

- Let $\mathcal{X} \subset \mathbb{R}^2$ be a region of the plane.
- Assume you have a *demand density* $g(x) : \mathcal{X} \to \mathbb{R}^+$ that you want to estimate.

Problem:
You can only have an estimate of:

$$y_i = \int_{V_i} g(x) dx$$

where V_i is a cell associated to measurement site s_i.

Example: s_i are the gas stations, you only have access to total demand, V_i is the "attraction region" of the gas station.

Spatial estimation

Radial basis functions

- We use Gaussian Radial Basis Functions to approximate $g(x)$.
- Using a sum of squares loss:

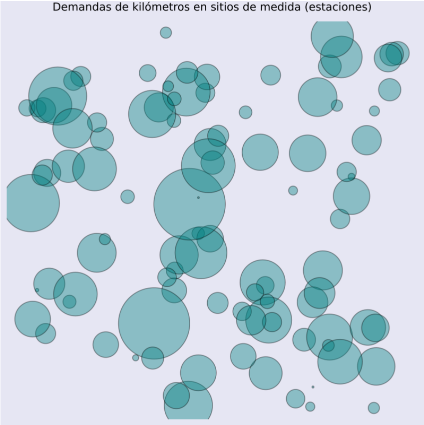
$$\mathcal{L} = \sum_i \left[\int_{V_i} g(x;\theta)\, dx - y_i \right]^2$$

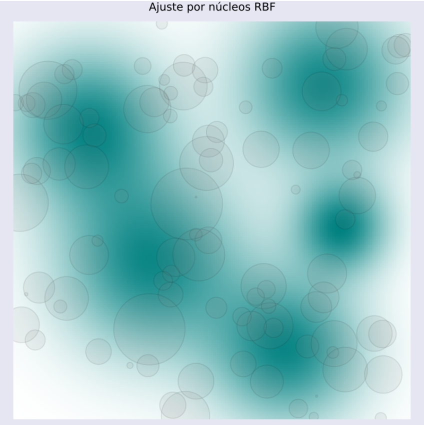
we can perform gradient descent.

- Nice Montecarlo trick to estimate the integrals, relates to stochastic gradient descent.

Spatial estimation

Radial basis functions





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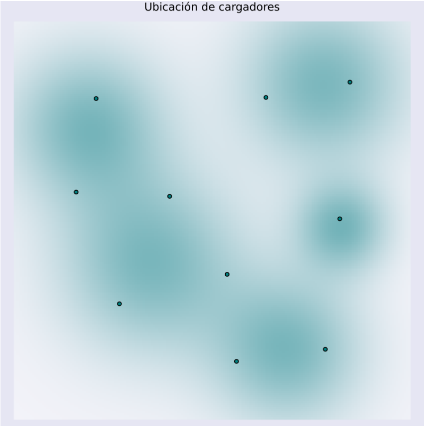
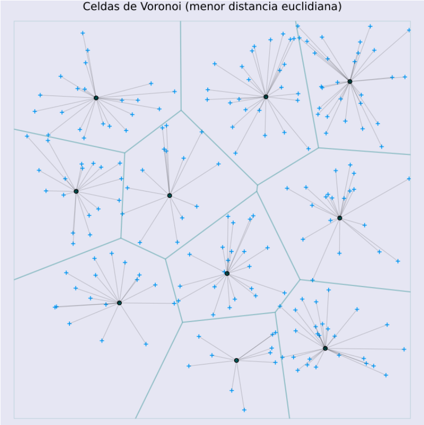
Radial basis functions





Spatial estimation

More detailed model

A second model distinguishes among requests:

- Requests come from a spatial Poisson process of intensity $\lambda(x)$.
- Each request needs some exponential amount of service of parameter ν.
- You only have access to:

$$y_i = \sum_n \sigma_n \mathbf{1}_{\{n \in V_i\}}$$

where σ_n are the (iid) individual requests.

Spatial estimation
Maximum likelihood approach

- For the second model, there is a *hidden variable* which is the number of requests.
- Using ideas from hidden Markov models, we can perform a maximum likelihood estimation of both $\lambda(x)$ and ν.
- For the $\lambda(x;\theta)$ we use RBFs again.

Outline of the talk

Take home messages...

- EVs are popping up, we have to prepare the infrastructure.
- We showed how a simple routing rule can handle congestion.
- Lagrangian duality decentralizes the problem using natural congestion signals.
- The algorithm is incentive compatible, so users' choices are natural.
- We also discussed how to estimate demand in order to build the correct infrastructure.

Thank you!

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